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Shape sensitivity analysis of sequential structural–acoustic problems using FEM and BEM

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Abstract

A shape design sensitivity formulation for structural–acoustic problems using sequential finite element and boundary element methods is presented. Frequency-response analysis is used to obtain the dynamic behavior of the structure, while boundary element analysis is used to solve for the pressure response of the acoustic domain. It is shown that the adjoint method, which takes the reverse direction to response analysis, provides a very efficient way of sensitivity calculation. In addition, it has been shown that the adjoint equation for the shape design problem is the same as that of the sizing design problem. The only difference is the numerical integration that evaluates the sensitivity coefficient. The combination of the semi-analytical method for the structure and the analytical differentiation method for the acoustic cavity yields a very practical approach for the shape design sensitivity formulation. The accuracy of the sensitivity information is compared with the analytical sensitivity as well as the sensitivity calculated using the finite difference method.

1. Introduction

This paper is the continuation of our previous publication [1] that treated the *sizing* design sensitivity formulation for sequential structural–acoustic problems. Here *shape* design sensitivity analysis of sequential structural–acoustic problems is presented in which the structural and

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acoustic behaviors are decoupled. When a harmonic excitation is applied, the dynamic behavior of the structure is described using frequency-response analysis. Boundary element analysis is then employed to calculate the radiated noise (pressure) from the structural response (harmonic velocity). This paper describes how to calculate the rate of change of the radiated noise effectively when the geometry of the structure is changed. The calculated sensitivity information can be used either in the interactive design process or in the automated optimization process. As sensitivity calculation is the most expensive process in optimization, the focus of the paper is on how to calculate it efficiently and accurately.

Traditionally, the sensitivity formulation can be classified into two approaches: the *direct method* and the *adjoint method*. The former calculates the sensitivity of the state response by differentiating the governing equation and then, using the chain rule of differentiation to calculate the performance sensitivity. The latter, however, calculates the performance sensitivity without recourse to the sensitivity of the state response. Rather, it utilizes the adjoint problem in calculating the implicitly dependent terms [2]. When the number of design variables is greater than that of performance functions, the adjoint method is more efficient than the direct method.

In the literature, many shape sensitivity formulations of structural–acoustic problems have been presented using the boundary element method. Most shape sensitivity formulations in the literature use the direct method. For example, Smith and Bernhard [3] presented the semi-analytical sensitivity formulation using the direct method. Koo et al. [4,5], Kane et al. [6], and Matsumoto et al. [7] derived the acoustic sensitivity expression with respect to the shape design variable. When only boundary element analysis is used [3–7], it is necessary that the velocity on the boundary and its shape sensitivity be prescribed.

Since the velocity on the boundary is determined through structural analysis, a finite element method and a boundary element method have been used sequentially in calculating the radiated noise [8]. The fundamental assumption is that the vibration of the structure is not affected by the bounding acoustic domain. Englestad et al. [9] optimized the interior noise problem using the direct method of sensitivity calculation. They considered sizing design variables; i.e., the thickness of the plate. Hahn and Ferri [10] used the perturbation technique to derive the sensitivity expression with respect to sizing design variables. Since the acoustic domain is independent of sizing design, they only perturbed the finite element matrix. Mallardo and Aliabadi [11] used the shape sensitivity information in order to identify the location of flaws by minimizing the error between the numerical and experimental data. As far as authors' knowledge extends, there is no publication available on the adjoint method in the shape sensitivity formulation of sequential structural–acoustic problems, which is the main purpose of this paper. The shape design sensitivity formulation is derived using the adjoint method that takes the reverse order of response analysis. It has been shown that the adjoint equation is identical for sizing and shape design problems. The proposed approach can also be applied to the acoustic problem that uses boundary element analysis only, if the surface velocity and its sensitivity are assumed to be available.

The composition of the paper is as follows. After a brief review of structural–acoustic analysis using the finite element and boundary element methods in Section 2, the shape sensitivity formulation is developed in Section 3 using direct and adjoint methods. Numerical examples are shown in Section 4 in order to show the accuracy and efficiency of the proposed sensitivity calculation method, followed by conclusions in Section 5. In order to simplify the presentation, the matrix notation is used as much as possible. However, the detailed derivation of shape sensitivity, which must be used the analytical form, is collected in the appendix.

2. Review of structural–acoustic analysis

In this section, a brief review of structural–acoustic analysis using the sequential finite element and boundary element methods is presented. This section is necessary in order to derive the sensitivity formulation in the following section. However, for a detailed explanation of the analysis procedure the reader is referred to the literature. The fundamental assumption is that the density of the acoustic medium is low compared to that of the structure so that the influence of the acoustic medium on the structure can be ignored. Thus, the structural–acoustic response is decoupled and the solution procedure is sequential. For a detailed explanation the reader is referred to Kim et al. [1].

2.1. Structural finite element analysis

A structural–acoustic system with a fully enclosed volume is shown in Fig. 1. All members of the structure are assumed to be plates and/or beams. The structure encloses a three-dimensional acoustic medium whose dynamic response is controlled by that of the structure. Let Ω^a and Ω^s be the domain of the acoustic medium and the structure, respectively. The acoustic domain has a boundary $\Gamma = \Gamma^{ar} \cup \Gamma^{as}$, where Γ^{ar} is the rigid boundary, and Γ^{as} is the structural boundary as the acoustic medium interfaces with the structure. Thus, $\Gamma^{as} = \Omega^s$. In addition, the structure has a boundary, Γ^s, that is not shared with acoustic medium. Even if an acoustic source can directly be applied to the acoustic cavity, it is assumed for simplicity that the excitation comes through the structural boundary.

When a harmonic load with the magnitude $\{\mathbf{f}\}$ and excitation frequency ω is applied to the structure, the steady-state response of the structure can be calculated from frequency-response

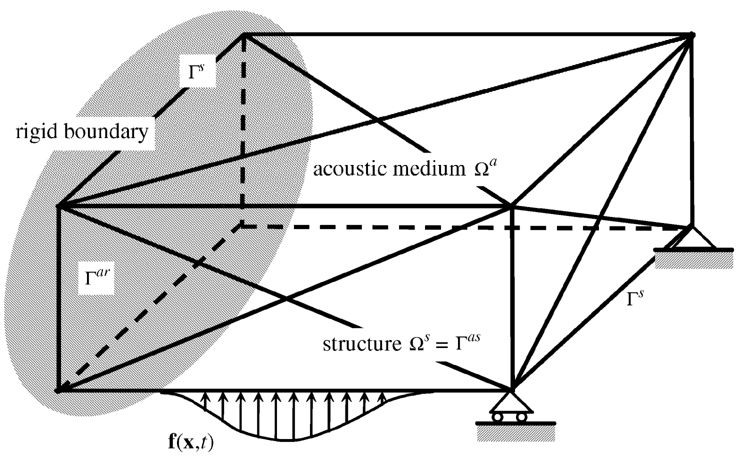

Fig. 1. Structural–acoustic system.

analysis. Let ϕ be the structural damping coefficient. Then, the structural velocity $\{\mathbf{v}(\omega)\}$ can be calculated using the following matrix equation:

$$[\mathrm{j}\omega\mathbf{M} + \kappa\mathbf{K}]\{\mathbf{v}(\omega)\} = \{\mathbf{f}(\omega)\}, \tag{1}$$

where $\mathrm{j} = \sqrt{-1}$, $[\mathbf{M}]$ is the mass matrix, $\mathbf{K}$ is the stiffness matrix, and $\kappa = (\phi - \mathrm{j})/\omega$. In this paper a system level matrix will be denoted by a bold typeface within brackets, whereas a system level vector will be denoted by a bold typeface within braces. Note that the steady-state response has the same frequency as the applied load but may have a different phase angle due to the existence of damping. In frequency-response analysis, all vectors and matrices are complex variables. Even if the applied load is not harmonic, Eq. (1) can still be applied by decomposing the time-dependent forces into the frequency domain and solving Eq. (1) at various frequencies.

2.2. Acoustic boundary element analysis

After calculating the velocity response of the structure, the boundary element method can be used to evaluate the pressure response within the acoustic domain. The standard wave equation is first reduced to the Helmholtz equation [12]. By integrating over the domain and by using Green's theorem, the Helmholtz equation constitutes the following boundary integral equation:

$$\alpha p(\mathbf{x}_0) = \iint_{\Gamma} \left[-\mathrm{j}\rho\omega G(\mathbf{x}_s, \mathbf{x}_0) v_n(\mathbf{x}_s) - \frac{\partial G}{\partial n} p(\mathbf{x}_s) \right] \mathrm{d}\Gamma, \tag{2}$$

where $G(\mathbf{x}_s, \mathbf{x}_0)$ is Green's function, $\mathbf{x}_s \in \Gamma$ is the position of a reference point, $\mathbf{x}_0$ is the position of an observation point, v_n is the normal component of the boundary velocity, and $\partial/\partial n$ is the normal component of the gradient. In Eq. (2), the constant α is equal to 1 for $\mathbf{x}_0$ inside the acoustic domain, 0.5 for $\mathbf{x}_0$ on the smooth boundary surface, and 0 for $\mathbf{x}_0$ outside the acoustic domain. Note that Eq. (2) can provide a solution for both radiation and interior acoustic problems.

The boundary element analysis procedure has two steps: first evaluation of the pressure response on the acoustic boundary using the structural velocity, and then calculation of the pressure response within the acoustic domain using the boundary pressure information. Let the acoustic boundary $\Gamma = \Gamma^{ar} \cup \Gamma^{as}$ be approximated by N number of nodes. If observation point $\mathbf{x}_0$ is repeatedly positioned at every node, then the following linear system of equations is obtained:

$$[\mathbf{A}]\{\mathbf{p}_s\} = [\mathbf{B}]\{\mathbf{v}\}, \tag{3}$$

where $\mathbf{p}_s = \{p_1, p_2, \ldots, p_N\}^{\mathrm{T}}$ is the nodal pressure vector, and $\{\mathbf{v}\}$ is the $3N \times 1$ velocity vector. For those nodes on the rigid boundary Γ^{ar}, the velocity vanishes. Matrices $[\mathbf{A}]_{(N\times N)}$ and $[\mathbf{B}]_{(N\times 3N)}$ are coefficient matrices calculated from the integrands of Eq. (2) at each node point. Note that these vectors and matrices are all complex variables. Once $\{\mathbf{p}_s\}$ has been computed, Eq. (2) can be used again to compute the acoustic pressure at any point $\mathbf{x}_0$ within the acoustic domain in the form of a vector equation as

$$p(\mathbf{x}_0) = \{\mathbf{b}(\mathbf{x}_0)\}^{\mathrm{T}}\{\mathbf{v}\} + \{\mathbf{e}(\mathbf{x}_0)\}^{\mathrm{T}}\{\mathbf{p}_s\}, \tag{4}$$

where $\{\mathbf{b}(\mathbf{x}_0)\}$ and $\{\mathbf{e}(\mathbf{x}_0)\}$ are the column vectors that correspond to the right-hand side of the boundary integral in Eq. (2).

3. Shape design sensitivity analysis

In the case of sizing design [1,13], the boundary integral equation does not contain any terms that are explicitly dependent on the design; only implicitly dependent terms exist through the state variables. In the shape design problem, however, the boundary integral equation is also changed when the enclosing structural domain is changed. In order to make sensitivity calculation more practical, a hybrid method is employed: the semi-analytical method for the finite element method and the analytical method for the boundary element method.

3.1. Shape design parameterization

In the proposed structural–acoustic problem, the structural domain is a part of the boundary of the acoustic domain. Thus, shape design parameterization changes the structural domain as well as the boundary of the acoustic domain. Since the detailed definition of shape parameterization is out of the scope of the paper, a brief sketch is provided. For a detailed definition users are referred to literature [14,15].

In this paper, we use the concept of design perturbation in representing shape design parameterization. Let us start with the assumption that for a given shape design variable the material point $\mathbf{x}$ of the structure moves in the direction of $\mathbf{\Theta}(\mathbf{x})$ (design perturbation). Let a scalar parameter τ denote the amount of shape change as shown in Fig. 2, then the new point $\mathbf{x}_\tau$ after design change can be expressed by

$$\mathbf{x}_\tau = \mathbf{x} + \tau\mathbf{\Theta}(\mathbf{x}), \quad \mathbf{x} \in \Omega^s. \tag{5}$$

When a part of the structure does not move according to the shape design, its design perturbation $\mathbf{\Theta}(\mathbf{x})$ is equal to zero. Eq. (5) provides a linear perturbation theory in which a linear design perturbation is used. If the scalar parameter τ is considered as time, then Eq. (5) is similar to the dynamic process, which is the reason that the design perturbation $\mathbf{\Theta}(\mathbf{x})$ is often called "design velocity".

Shape sensitivity of a function $g(\mathbf{x})$ is the rate of change of $g(\mathbf{x})$ in the direction of design perturbation $\mathbf{\Theta}(\mathbf{x})$. In the shape design problem, this change includes the change in function value as well as the change caused by the change in the location, which is similar to the total derivative in mechanics. Thus, the shape sensitivity of a function $g(\mathbf{x})$ can be written as

$$\dot{g}(\mathbf{x}) \equiv \frac{\mathrm{d}}{\mathrm{d}\tau} g(\mathbf{x} + \tau\mathbf{\Theta})|_{\tau=0} = \lim_{\tau\to 0}\left[\frac{g_\tau(\mathbf{x} + \tau\mathbf{\Theta}) - g(\mathbf{x})}{\tau}\right]. \tag{6}$$

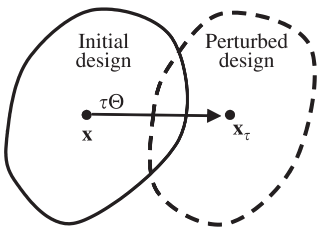

Fig. 2. Shape design perturbation. The initial domain is changed to the perturbed domain according to design perturbation $\mathbf{\Theta}(\mathbf{x})$.

The first step of shape design is to obtain the design perturbation corresponding to the shape design variable. Then, the sensitivity formulation in the following subsection expresses the performance sensitivity in terms of the design perturbation.

3.2. Direct method

The direct method of sensitivity formulation is to first calculate the sensitivity of the state variable and then to use the chain rule of differentiation to calculate the sensitivity of the performance function. This method is popular because it is closely related to the analysis procedure. First, the finite element matrix equation (1) is differentiated with respect to the shape design as

$$[\mathrm{j}\omega\mathbf{M} + \kappa\mathbf{K}]\{\dot{\mathbf{v}}(\omega)\} = \{\dot{\mathbf{f}}(\omega)\} - [\mathrm{j}\omega\dot{\mathbf{M}} + \kappa\dot{\mathbf{K}}]\{\mathbf{v}(\omega)\}, \tag{7}$$

where the solution $\{\dot{\mathbf{v}}(\omega)\}$ is desired for a given excitation frequency ω. In Eq. (7), the superposed "dot" represents the shape sensitivity as in Eq. (6). When the design perturbation is given, the right-hand side of Eq. (7) can be calculated using the structural solution $\{\mathbf{v}(\omega)\}$ and variations of the force vector and mass and stiffness matrices.

The vector and matrices on the right-hand side of Eq. (7) can be expressed analytically if the continuum form is used [16]. However, their expressions can be complicated, especially when the structural domain is approximated using shell finite elements. The element experiences shape change as well as orientation change. A compromised approach would be the semi-analytical derivative, in which the variation of the coefficient matrices can be calculated using the finite difference method. For example, let the element mass matrix be denoted by $[\mathbf{m}(\mathbf{x})]$. Then, the variation of the mass matrix can be calculated using a small perturbation τ, as

$$[\dot{\mathbf{m}}] \approx \frac{[\mathbf{m}(\mathbf{x}_\tau)] - [\mathbf{m}(\mathbf{x})]}{\tau}. \tag{8}$$

The same approach can be used for the stiffness matrix and the force vector. This semi-analytical method is in fact implemented into the MSC/NASTRAN finite element analysis program [17], which will be used in the numerical example section. When analytical variations for the mass and stiffness matrices are available, the proposed method can also be used without any modifications.

After calculating the sensitivity $\{\dot{\mathbf{v}}(\omega)\}$ of the boundary velocity, the sensitivity of the surface pressure needs to be calculated by differentiating the boundary element matrix equation (3). By following a procedure similar to the finite element method, the sensitivity equation for the boundary element method can be obtained as

$$[\mathbf{A}]\{\dot{\mathbf{p}}_s\} = [\mathbf{B}]\{\dot{\mathbf{v}}\} + [\dot{\mathbf{B}}]\{\mathbf{v}\} - [\dot{\mathbf{A}}]\{\mathbf{p}_s\}, \tag{9}$$

where the solution $\{\dot{\mathbf{p}}_s\}$ is required. The first term on the right-hand side can be calculated using $\{\dot{\mathbf{v}}(\omega)\}$ from Eq. (7), and the other terms can be calculated from the given design perturbation $\boldsymbol{\Theta}(\mathbf{x})$. In the case of those matrices that appear in boundary element analysis, the semi-analytical method will be expensive since the matrices are full and un-symmetric. Thus, the analytical expressions of their variations are developed in the appendix.

Using $\{\dot{\mathbf{v}}(\omega)\}$ and $\{\dot{\mathbf{p}}_s\}$ on the acoustic boundary, the pressure sensitivity at the observation point $\mathbf{x}_0$ interior (or exterior) to the acoustic domain can be evaluated from Eq. (4), as

$$\dot{p}(\mathbf{x}_0) = \{\dot{\mathbf{e}}(\mathbf{x}_0)\}^{\mathrm{T}}\{\mathbf{p}_s\} + \{\mathbf{e}(\mathbf{x}_0)\}^{\mathrm{T}}\{\dot{\mathbf{p}}_s\} + \{\dot{\mathbf{b}}(\mathbf{x}_0)\}^{\mathrm{T}}\{\mathbf{v}\} + \{\mathbf{b}(\mathbf{x}_0)\}^{\mathrm{T}}\{\dot{\mathbf{v}}\}. \tag{10}$$

The expression of $\{\dot{\mathbf{b}}(\mathbf{x}_0)\}$ and $\{\dot{\mathbf{e}}(\mathbf{x}_0)\}$ can be calculated using the same method above, as explained in the appendix. After calculating $\dot{p}(\mathbf{x}_0)$, the sensitivity of general performance can be calculated using the chain rule of differentiation.

As mentioned before, the direct method is closely related to the analysis procedure. Each design variable requires the design perturbation. Since the sensitivity equations (7) and (9) depend on the design perturbation, they need to be solved for the same number of design variables, which constitutes the main computational cost of sensitivity analysis. However, since the coefficient matrices in Eqs. (7) and (9) are already factorized during the response analysis, the computational cost of sensitivity analysis is relatively inexpensive.

3.3. Adjoint method

Different from the direct method, the adjoint method is closely related to the performance function rather than response analysis. The basic idea of the adjoint method is to avoid calculating the sensitivity of the response variables (i.e., $\{\dot{\mathbf{v}}\}$ and $\{\dot{\mathbf{p}}_s\}$), as they need to be calculated per each design variable. In order to start, consider an acoustic pressure at point $\mathbf{x}_0$ as a performance function, defined as

$$\psi = p(\mathbf{x}_0). \tag{11}$$

It is assumed that the observation point $\mathbf{x}_0$ in Eq. (11) is independent of the design; i.e., it is a fixed point in the acoustic domain. Note that the acoustic performance function is defined at a point, which is different from the structural performance that is defined as an integral form [1]. The sensitivity of the acoustic performance can then be obtained from Eqs. (9) and (10), as

$$\begin{aligned}\dot{\psi} &= \{\mathbf{e}\}^{\mathrm{T}}\{\dot{\mathbf{p}}_s\} + \{\mathbf{b}\}^{\mathrm{T}}\{\dot{\mathbf{v}}\} + \{\dot{\mathbf{e}}\}^{\mathrm{T}}\{\mathbf{p}_s\} + \{\dot{\mathbf{b}}\}^{\mathrm{T}}\{\mathbf{v}\} \\ &= \{\mathbf{e}\}^{\mathrm{T}}[\mathbf{A}]^{-1}\{[\mathbf{B}]\{\dot{\mathbf{v}}\} + [\dot{\mathbf{B}}]\{\mathbf{v}\} - [\dot{\mathbf{A}}]\{\mathbf{p}_s\}\} + \{\mathbf{b}\}^{\mathrm{T}}\{\dot{\mathbf{v}}\} + \{\dot{\mathbf{e}}\}^{\mathrm{T}}\{\mathbf{p}_s\} + \{\dot{\mathbf{b}}\}^{\mathrm{T}}\{\mathbf{v}\}.\end{aligned} \tag{12}$$

Thus, the variation of the performance function is expressed in terms of the variation of the structural velocity in Eq. (12); i.e., the calculation of $\{\dot{\mathbf{p}}_s\}$ is avoided. In Eq. (12), the calculation of $[\mathbf{A}]^{-1}$ is expensive and unnecessary. Instead, consider the following form of the acoustic adjoint equation:

$$[\mathbf{A}]^{\mathrm{T}}\{\boldsymbol{\eta}\} = \{\mathbf{e}\}, \tag{13}$$

where the acoustic adjoint solution $\{\boldsymbol{\eta}\}$ is desired. Then, the solution of Eq. (13) can be expressed as $\{\boldsymbol{\eta}\}^{\mathrm{T}} = \{\mathbf{e}\}^{\mathrm{T}}[\mathbf{A}]^{-1}$, which is the same as the matrix inverse part in Eq. (12). Note that the acoustic adjoint equation (13) is independent of the design. Thus, the adjoint equation needs to be defined once for all design variables.

After substituting $\{\boldsymbol{\eta}\}^{\mathrm{T}}$ into the right-hand side of Eq. (12) and after replacing the relation in Eq. (7), the dependence on $\{\dot{\mathbf{v}}\}$ is removed in the sensitivity expression of the performance, as

$$\begin{aligned}\dot{\psi} &= \{[\mathbf{B}]^{\mathrm{T}}\{\boldsymbol{\eta}\} + \{\mathbf{b}\}\}^{\mathrm{T}}\{\dot{\mathbf{v}}\} + \{\boldsymbol{\eta}\}^{\mathrm{T}}\{[\dot{\mathbf{B}}]\{\mathbf{v}\} - [\dot{\mathbf{A}}]\{\mathbf{p}_s\}\} + \{\dot{\mathbf{e}}\}^{\mathrm{T}}\{\mathbf{p}_s\} + \{\dot{\mathbf{b}}\}^{\mathrm{T}}\{\mathbf{v}\} \\ &= \{[\mathbf{B}]^{\mathrm{T}}\{\boldsymbol{\eta}\} + \{\mathbf{b}\}\}^{\mathrm{T}}[\mathrm{j}\omega\mathbf{M} + \kappa\mathbf{K}]^{-1}\{\dot{\mathbf{f}} - [\mathrm{j}\omega\dot{\mathbf{M}} + \kappa\dot{\mathbf{K}}]\{\mathbf{v}\}\} \\ &\quad + \{\boldsymbol{\eta}\}^{\mathrm{T}}\{[\dot{\mathbf{B}}]\{\mathbf{v}\} - [\dot{\mathbf{A}}]\{\mathbf{p}_s\}\} + \{\dot{\mathbf{e}}\}^{\mathrm{T}}\{\mathbf{p}_s\} + \{\dot{\mathbf{b}}\}^{\mathrm{T}}\{\mathbf{v}\}.\end{aligned} \tag{14}$$

By following the same procedure as Eq. (13), the matrix inverse part can be removed by defining the following form of the structural adjoint equation:

$$[\mathrm{j}\omega\mathbf{M} + \kappa\mathbf{K}]^{\mathrm{T}}\{\boldsymbol{\lambda}^*\} = \{\mathbf{b}\} + [\mathbf{B}]^{\mathrm{T}}\{\boldsymbol{\eta}\}, \tag{15}$$

where the structural adjoint solution $\{\boldsymbol{\lambda}^*\}$ is desired. The right superscript $*$ denotes that the variable is a complex conjugate. Since the mass and stiffness matrices are symmetric, the coefficient matrix on the left-hand side is the same as that of Eq. (1). Again, the structural adjoint equation (15) is independent of design. Thus, the adjoint equation needs to be defined once for all design variables. After substituting the structural adjoint solution to Eq. (14), the sensitivity expression of ψ can be obtained as

$$\dot{\psi} = \{\boldsymbol{\lambda}^*\}^{\mathrm{T}}\{\dot{\mathbf{f}} - \mathrm{j}\omega[\dot{\mathbf{M}}]\{\mathbf{v}\} - \kappa[\dot{\mathbf{K}}]\{\mathbf{v}\}\} + \{\boldsymbol{\eta}\}^{\mathrm{T}}[\dot{\mathbf{B}}]\{\mathbf{v}\} - \{\boldsymbol{\eta}\}^{\mathrm{T}}[\dot{\mathbf{A}}]\{\mathbf{p}_s\} + \{\dot{\mathbf{e}}\}^{\mathrm{T}}\{\mathbf{p}_s\} + \{\dot{\mathbf{b}}\}^{\mathrm{T}}\{\mathbf{v}\}. \tag{16}$$

Note that the acoustic adjoint equation (13) and the structural adjoint equation (15) are exactly the same as that of the sizing design problem presented by Kim et al. [1]. Thus, no additional effort is required for calculating the adjoint solutions. The only difference is calculating the sensitivity coefficient in Eq. (16). In the sizing design problem, the last four terms in Eq. (16) do not exist because the acoustic domain is independent of the sizing design variable. However, in the case of the shape design, the design change causes the change of the acoustic domain whose contribution is basically the last four terms Eq. (16).

It is noted from Eqs. (13) and (15) that the adjoint equations need to be solved once for all design variables and the sensitivity expression in Eq. (16) is repeated for all design variables. This makes the proposed method very efficient compared with the direct method when the number of performance functions is smaller than that of design variables, which is the case for most optimization problems.

As can be seen in Eqs. (13) and (15), the solution procedure of the adjoint method is the opposite to the response analysis procedure: the acoustic adjoint equation is solved first and then, the structural adjoint equation is solved. This can be a reason that the adjoint method is not popular in structural–acoustic sensitivity analysis. However, the procedure proposed in Eqs. (13)–(16) is simple and straightforward. Another possible reason is that the boundary element coefficient matrix is unsymmetric. Thus, the boundary element matrix equation (3) is different from the acoustic adjoint equation (13). Even if the coefficient matrix is not symmetric, the factorized coefficient matrix from one analysis can still be used for the transposed system without causing further computational cost. In summary, the proposed adjoint method does not require any more information than the direct method, but can be very efficient when the number of performance functions is smaller than that of shape design variables.

4. Numerical examples

Three numerical examples are presented. The purpose of the first example is the validation of the sensitivity calculation routines related to the boundary element formulation. The second example contains both the finite element and boundary element methods. However, only the shape of the boundary elements is changed. The third example uses the full capability of the proposed sensitivity calculation method.

4.1. Analytical validation of sensitivity results using BEM

The first example is presented to show the sensitivity accuracy of the boundary element method alone. To that end, the radiated pressure from a sphere with a constant radial velocity is calculated using boundary element analysis. Fig. 3 shows the boundary element model of a sphere with a radius of $a = 1\,\mathrm{m}$, and a constant radial velocity of $v = 1\,\mathrm{m/s}$ is applied at each node. The sphere is discretized using 384 quadrilateral elements and 386 nodes. The following material properties are used for air: the density $\rho = 1.205\,\mathrm{kg/m^3}$ and the speed of sound $c = 344\,\mathrm{m/s}$. The analytical solution to the pressure at a distance r from the center of the sphere can be found in Ref. [18] as

$$p(r) = \frac{\mathrm{j}ka^2\rho cv}{1 + \mathrm{j}ka}\frac{\mathrm{e}^{-\mathrm{j}k(r-a)}}{r}. \tag{17}$$

The radiated pressure at the position (6, 0, 0) is evaluated at various frequency ranges between 1 and 100 Hz. In order to resolve the singularity problem of the exterior acoustic analysis, the over-determination method [19] is employed by choosing an interior point (0, 0, 0) as an additional observation point.

In this simple example, the radius of the sphere is chosen as a design variable. As can be seen in the expression of Eq. (17), the pressure will be changed according to the shape design. The design sensitivity is the rate of pressure change with respect to radius change. Note that the radial velocity at the surface remains constant and thus, the sensitivity of the velocity is equal to zero. For design sensitivity analysis, the design perturbation is first defined, which is the unit normal

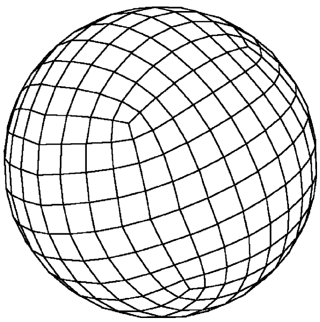

Fig. 3. Boundary elements for the acoustic sphere model.

vector to the surface. The design sensitivity equation (9) can be simplified as

$$[\mathbf{A}]\{\dot{\mathbf{p}}_s\} = [\dot{\mathbf{B}}]\{\mathbf{v}\} - \{\dot{\mathbf{A}}]\{\mathbf{p}_s\}. \tag{18}$$

After calculating the variation of the surface pressure, the variation of the pressure can be evaluated from Eq. (10) as

$$\dot{p}(r) = \{\mathbf{e}(r)\}^{\mathrm{T}}\{\dot{\mathbf{p}}_s\} + \{\dot{\mathbf{e}}(r)\}^{\mathrm{T}}\{\mathbf{p}_s\} + \{\dot{\mathbf{b}}(r)\}^{\mathrm{T}}\{\mathbf{v}\}. \tag{19}$$

In the adjoint method, the explicit calculation of $\{\dot{\mathbf{p}}_s\}$ is avoided by adopting the following form of the adjoint equation:

$$[\mathbf{A}]^{\mathrm{T}}\{\boldsymbol{\eta}\} = \{\mathbf{b}\}, \tag{20}$$

where the acoustic adjoint response $\{\boldsymbol{\eta}\}$ is expected. After solving for the acoustic adjoint response, the variation of the pressure can be calculated as

$$\dot{p}(r) = \{\boldsymbol{\eta}\}^{\mathrm{T}}[\dot{\mathbf{B}}]\{\mathbf{v}\} - \{\boldsymbol{\eta}\}^{\mathrm{T}}[\dot{\mathbf{A}}]\{\mathbf{p}_s\} + \{\dot{\mathbf{e}}(r)\}\{\mathbf{p}_s\} + \{\dot{\mathbf{b}}(r)\}\{\mathbf{v}\}. \tag{21}$$

Note that the same discussion regarding the efficiency of the adjoint variable method is also applied for the acoustic problem.

In order to show the accuracy of the proposed sensitivity calculation, the sensitivity result obtained from boundary element analysis is compared with the sensitivity result obtained by directly differentiating the analytical solution in Eq. (17). The analytical sensitivity expression is

$$\dot{p}(r) = \frac{\rho c v k a[(2\mathrm{j} - ka)(1 + \mathrm{j}ka) + ka]}{r(1 + \mathrm{j}ka)^2}\mathrm{e}^{-\mathrm{j}k(r-a)}\delta a. \tag{22}$$

Fig. 4 shows the graph of $\dot{p}(r)$ at $r = 6$ with various frequency ranges between 1 and 100 Hz. It is clear that the sensitivity result obtained from the proposed method is well matched with that from the analytical sensitivity.

This model verifies the accuracy of the sensitivity analysis using a boundary element method only. This procedure is possible because the velocity on the surface is assumed to be constant.

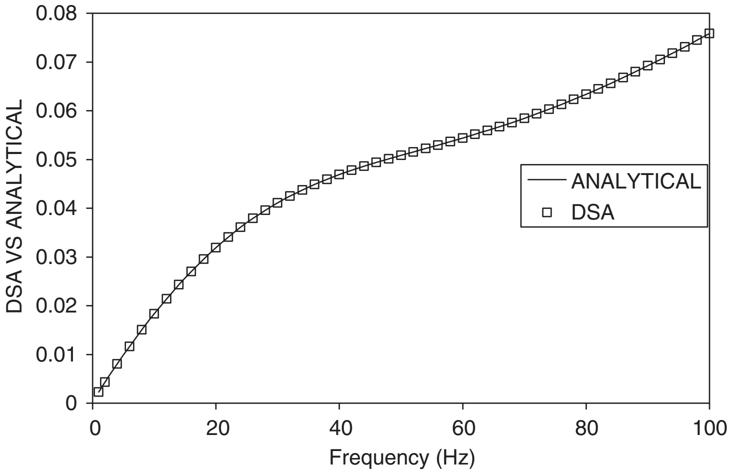

Fig. 4. Design sensitivity results at position (6, 0, 0).

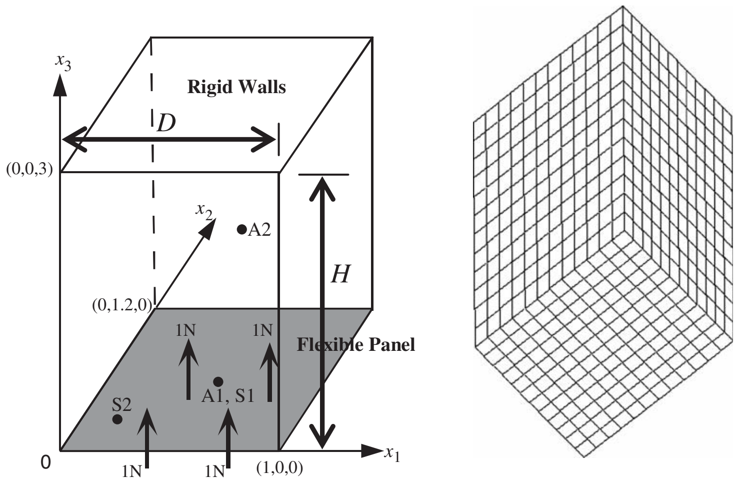

Fig. 5. Acoustic box model. Five sides are rigid walls and the bottom plate is flexible.

However, in real applications the velocity also depends on the shape design variable. Thus, it is necessary to verify the sensitivity results when the velocity changes according to the design, which is demonstrated in the following section.

4.2. Shape change in the acoustic domain

Consider an acoustic box covered by five rigid walls and one flexible panel on the bottom, as illustrated in Fig. 5. The goal is to determine how the pressure response changes according to the shape design variable that changes the height of the acoustic cavity (H in Fig. 5). In this case, the design perturbation in Eq. (5) can be written as

$$\boldsymbol{\Theta}(\mathbf{x}) = \tfrac{1}{3} x_3 \mathbf{k}, \tag{23}$$

where $\mathbf{k}$ is the unit vector in the x_3-coordinate direction. The maximum magnitude of the design perturbation is equal to one at the top surface. Note that the design perturbations for all structural nodes are zero because their x_3-coordinates are zero. Only the acoustic nodes have non-zero design perturbations.

The same properties as in the sphere model are used for the acoustic medium. The panel is made of an aluminum plate with a thickness of $t = 0.01\,\mathrm{m}$, mass density of $\rho_s = 2700\,\mathrm{kg/m^3}$, Young's modulus of $E = 7.1 \times 10^{10}\,\mathrm{Pa}$, Poisson's ratio of $v = 0.334$, and a structural–damping coefficient of $\phi = 0.06$. A harmonic force with a magnitude of $f = 1.0\,\mathrm{N}$ in the x_3-direction is applied at four points on the plate as shown in the figure. The whole structure is discretized by 144 finite elements (flexible panel) and 864 boundary element (rigid walls).

The following pressure performance functions are considered: the acoustic pressure at A_1 (0.5, 0.6, 0), the interface point at the flexible panel center; and at A_2 (0.5, 0.6, 1.5), the center of the cavity. The MSC/NASTRAN program [17] is used for frequency-response analysis, whereas the boundary element analysis is used for evaluating the acoustic pressure in the cavity. Fig. 6 provides the amplitude of pressure at points A_1 and A_2 for the frequency range between 1 and

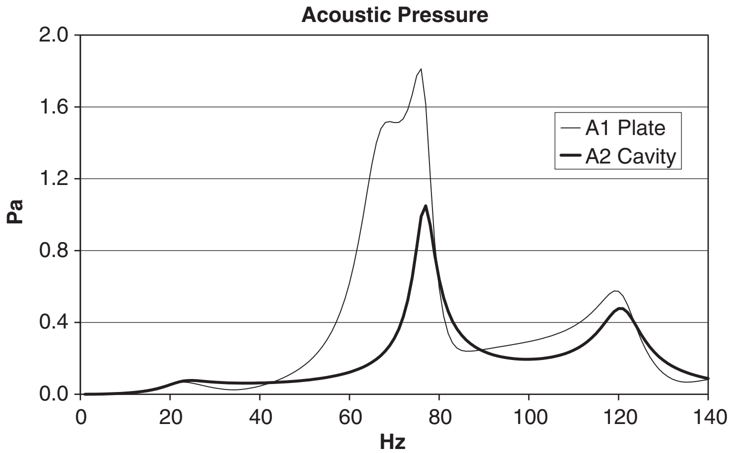

Fig. 6. Pressure results of the acoustic box at the plate center and cavity center.

140 Hz. The peak values of the pressure appear at frequencies corresponding to natural frequencies of the plate.

In this particular example, the structural domain (bottom plate) is independent of the design, whereas the acoustic domain is changed according to the design. Thus, there is no explicitly dependent term in the structural sensitivity equation, which means the right-hand side of Eq. (7) vanishes. Thus, it is unnecessary to perform the structural adjoint equation and the sensitivity expression of ψ becomes

$$\dot{\psi} = \{\boldsymbol{\eta}\}^{\mathrm{T}}\{[\dot{\mathbf{B}}]\{\mathbf{v}\} - [\dot{\mathbf{A}}]\{\mathbf{p}_s\}\} + \{\dot{\mathbf{e}}\}^{\mathrm{T}}\{\mathbf{p}_s\} + \{\dot{\mathbf{b}}\}^{\mathrm{T}}\{\mathbf{v}\}, \tag{24}$$

which includes the explicitly dependent terms from boundary element analysis. In Eq. (24), the structural velocity $\{\mathbf{v}\}$ from Eq. (1), acoustic pressure from Eq. (3), and the adjoint solution $\{\boldsymbol{\eta}\}$ from Eq. (13) are used in evaluating the sensitivity expression.

As shown in Fig. 5, the height H of the acoustic cavity is selected as a shape design variable. The design sensitivities are computed at 76 Hz, which is close to the resonant frequency, as shown in Fig. 6. The design sensitivity results are shown in Table 1. Since the pressure $p = p_r + \mathrm{j}p_i$ is a complex variable, the sensitivity of its amplitude can be calculated from the following formula:

$$|p|^{\cdot} = \frac{p_r\dot{p}_r + p_i\dot{p}_i}{|p|}, \tag{25}$$

where $\dot{p} = \dot{p}_r + \mathrm{j}\dot{p}_i$ is obtained from the design sensitivity analysis.

Different from the sphere problem, no analytical solution is available for the box model. Thus, the best way of validating the accuracy of the sensitivity results is to compare them with those obtained from the finite difference method. In Table 1, $\psi(u)$ and $\psi(u+\Delta u)$ are the responses at designs u and $u+\Delta u$, respectively, where Δu is the amount of design perturbation. The forward finite difference design sensitivity is obtained by $\Delta\psi/\Delta u = [\psi(u+\Delta u) - \psi(u)]/\Delta u$, and $\dot{\psi}$ is the predicted design sensitivity using the proposed method. A design perturbation of $\Delta u = 1.0 \times 10^{-3}$ m is used, and the predicted values are compared with the finite difference results. Table 1

Table 1
Sensitivity accuracy compared to the finite difference method

Performance	$\psi(u)$	$\psi(u+\Delta u)$	$\Delta\psi/\Delta u$	$\dot{\psi}$	Ratio
Pressure at A_1	1.81181	1.80530	−6.507	−6.519	100.18
Pressure at A_2	0.99016	0.99211	1.941	1.943	100.10

Shape design is the height (H) of the acoustic domain.

presents design sensitivity results for the acoustic pressure in Pascal (Pa). Good agreement is obtained between $\dot{\psi}$ and $\Delta\psi/\Delta u$. The sensitivity results show that the pressure at the center of the bottom surface (Point A_1) is reduced, while the pressure at the center of the acoustic domain is increased (Point A_2).

In this example, the advantage of the adjoint variable method is not clearly shown, because the number of design variable is less than the number of performance measures. The efficiency of the method, however, has been shown in Ref. [1], in which the example has 144 design variables but one performance measure. If the direct differentiation method is employed in such a problem, then 144 design sensitivity equations must be solved, while with the adjoint variable method only one adjoint equation needs to be solved.

4.3. Shape change in the structural domain

The previous two examples do not perturb the structural domain. Thus, there is no need to calculate the structural adjoint response in Eq. (15). For example, the sensitivity expression in Eq. (24) only includes the acoustic adjoint solution $\{\boldsymbol{\eta}\}$ from Eq. (13). In this section, the same box model as in Fig. 5 is considered as a last example. In this case, however, the shape design variable is chosen such that the length of the flexible plate (D in Fig. 5) is changed. For such a design, the design perturbation is given as

$$\boldsymbol{\Theta}(\mathbf{x}) = x_1\mathbf{i}, \tag{26}$$

where $\mathbf{i}$ is the unit vector in the x_1-coordinate direction. This design variable changes both the structural and acoustic domains.

After solving the structural response $\{\mathbf{v}\}$ and the acoustic response $\{\mathbf{p}_s\}$, the acoustic adjoint problem in Eq. (13) and the structural adjoint problem in Eq. (15) are solved. The pressure sensitivities at points A_1 and A_2 are calculated using Eq. (16). Table 2 compares the sensitivity results obtained from the proposed method with those from the finite difference method. The same perturbation size as in the previous example is used for the finite difference calculation. Instead of comparing the magnitude of the pressure, the real part (R) and the imaginary part (I) are compared. Good agreement is observed between $\dot{\psi}$ (results from the proposed sensitivity calculation method) and $\Delta\psi/\Delta u$ (results from the finite difference method). It is noted that the sensitivity values in Table 2 are greater than those in Table 1, which means that the structural shape design changes the acoustic pressure more significantly than the acoustic design.

Table 2
Sensitivity accuracy compared to the finite difference method

Performance		$\psi(u)$	$\psi(u+\Delta u)$	$\Delta\psi/\Delta u$	$\dot{\psi}$	Ratio (%)
Pressure	R	1.7696	1.7738	4.1875	4.2019	100.34
at A_1	I	−0.3890	−0.4622	−73.205	−73.294	100.12
Pressure	R	0.2997	0.2664	−33.274	−33.271	99.99
at A_2	I	−0.9437	−0.9667	−22.933	−22.932	100.00

Shape design is the length (D) of the structural domain.

5. Conclusions and discussion

Shape sensitivity analysis of the sequential structural–acoustic problem is presented using the adjoint variable method. The reverse procedure of sensitivity analysis makes it possible to calculate the adjoint responses. It has been shown that the non-symmetric property of the acoustic boundary element formulation does not prohibit the use of the adjoint variable method. Due to the un-coupled nature of the problem, the structural adjoint problem is only required when the structural domain is changed according to the design perturbation. The proposed method shows promising results in accurately calculating sensitivity information using the adjoint variable method.

As has been shown by Kim et al. [1] and Dong et al. [13], the peak value of the acoustic noise usually occurs near the natural frequency of the acoustic domain. In the sizing design problem, however, the natural frequency of the acoustic domain remains constant because the shape of the acoustic domain does not change. Thus, it is unnecessary to consider shifting the peak value of the pressure during design optimization. However, in the shape design problem the shape of the acoustic domain indeed changes according to the design variable. Thus, careful consideration is required for selecting the performance function. Otherwise, the peak value of the pressure simply moves to another frequency without actually reducing the noise level.

Appendix

In this section, the shape variation of the boundary integral equation (2) is derived. Fig. 7 shows a quadrilateral boundary element with given nodal design perturbation vectors. It is assumed that all vectors are represented in the element-fixed local coordinates, including the design perturbation vectors. As the value of the design variable changes, the element geometry is moved to the perturbed element. Accordingly, the area of the element, as well as the orientation of the element is changed.

Consider the fundamental solution of the Helmholtz equation. In the three-dimensional problem, $G(\mathbf{x}_s, \mathbf{x})$ is defined as

$$G(\mathbf{x}_s, \mathbf{x}) = \frac{\mathrm{e}^{-\mathrm{j}kr}}{4\pi r}, \tag{27}$$

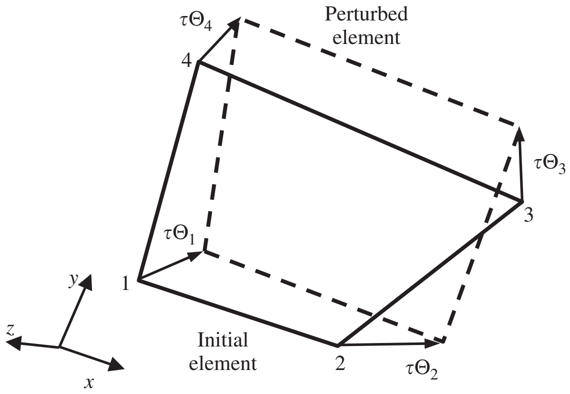

Fig. 7. Initial and perturbed boundary elements. Design perturbation vectors are defined at each node.

where $\mathbf{r} = \mathbf{x}_s - \mathbf{x}$, $r = \|\mathbf{r}\|$ and $k = \omega/c$ is the wavenumber. A bold typeface denotes a geometric vector. As the shape changes, the distance between the reference and observation points is changed. According to the design perturbation given in Eq. (5), the variation of this distance can be obtained as

$$\dot{r} = \frac{1}{r}\mathbf{r} \cdot (\mathbf{\Theta}_s - \mathbf{\Theta}), \tag{28}$$

where $\mathbf{\Theta}_s$ and $\mathbf{\Theta}$ are the design perturbation vectors at points $\mathbf{x}_s$ and $\mathbf{x}$, respectively. When the observation point is within the acoustic domain, $\mathbf{x}$ is independent of the design; i.e., $\mathbf{\Theta} = \mathbf{0}$. Using Eq. (28), the variation of the fundamental solution can be calculated, as

$$\dot{G}(\mathbf{x}_s, \mathbf{x}) = -\frac{\mathrm{e}^{-\mathrm{j}kr}}{4\pi r^2}(1 + \mathrm{j}kr)\dot{r}. \tag{29}$$

Note that the variations in Eqs. (28) and (29) explicitly depend on the design. Thus, for the given design perturbation these expressions can be obtained easily.

Next, as the shape of the structure changes, the normal vector to the boundary also changes its direction. From Fig. 7, let $\mathbf{a} = (\mathbf{x}_3 - \mathbf{x}_1) \times (\mathbf{x}_4 - \mathbf{x}_2)$ be the normal vector to the element. Then, the unit outward normal vector is defined as

$$\mathbf{n} = \frac{\mathbf{a}}{\|\mathbf{a}\|}, \tag{30}$$

whose variation can be obtained by

$$\dot{\mathbf{n}} = \frac{1}{\|\mathbf{a}\|}\left[\dot{\mathbf{a}} - (\dot{\mathbf{a}}^{\mathrm{T}}\mathbf{n})\mathbf{n}\right], \tag{31}$$

where $\dot{\mathbf{a}} = (\mathbf{\Theta}_3 - \mathbf{\Theta}_1) \times (\mathbf{x}_4 - \mathbf{x}_2) + (\mathbf{x}_3 - \mathbf{x}_1) \times (\mathbf{\Theta}_4 - \mathbf{\Theta}_2)$. Note that $\dot{\mathbf{n}}$ is the tangential component of $\dot{\mathbf{a}}$, normalized by $\|\mathbf{a}\|$.

In addition to the integrands, the integral domain Ω^s also depends on the shape design. For a given design perturbation in Eq. (5), the perturbed domain Ω^s_τ satisfies the following relation:

$$\mathrm{d}\Omega^s_\tau = J\,\mathrm{d}\Omega^s, \tag{32}$$

where $J = |\mathrm{d}\mathbf{x}_\tau/\mathrm{d}\mathbf{x}|$ is the determinant of the Jacobian relation. Thus, it is enough to consider the variation of J, which can be derived as

$$\dot{J} = \sum_{i=1}^{2} \frac{\partial \Theta_i}{\partial x_i}. \tag{33}$$

It is assumed that the structural domain (or acoustic boundary) is composed of NE number of boundary elements. Let us consider the variation of the first integrand on the right-hand side of Eq. (2). Using Eqs. (29), (31) and (33), the explicitly dependent terms can be obtained as

$$\{\dot{\mathbf{b}}\}^{\mathrm{T}}\{\mathbf{v}\} = \sum_{e=1}^{\mathrm{NE}} \iint_{\Omega_e^s} [-\mathrm{j}\rho\omega(\dot{G}\mathbf{n}\cdot\mathbf{v} + G\dot{\mathbf{n}}\cdot\mathbf{v} + \dot{J}G\mathbf{n}\cdot\mathbf{v})]\,\mathrm{d}\Omega_e^s. \tag{34}$$

For a given analysis result $\{\mathbf{v}\}$ and the design perturbation, the right-hand side of Eq. (34) can readily be evaluated using the same numerical integration method as boundary element analysis. As can be seen from Eq. (34), it is unnecessary to construct the global vector $\{\dot{\mathbf{b}}\}$. Rather, the right-hand side of Eq. (34) needs to be calculated and summed for each boundary element. In order to calculate $[\dot{\mathbf{B}}]\{\mathbf{v}\}$ in Eq. (16), the expression in Eq. (34) is repeated by setting the observation point to the location of all boundary nodes.

The second integrand of Eq. (2) can also be calculated using a similar method. The second fundamental solution to the Helmholtz equation is defined as

$$H(\mathbf{x}_s, \mathbf{x}) = \frac{\partial G(\mathbf{x}_s, \mathbf{x})}{\partial n} = -\frac{1 + \mathrm{j}kr}{4\pi r^3}\,\mathrm{e}^{-\mathrm{j}kr} r_n, \tag{35}$$

where $r_n = \mathbf{r}\cdot\mathbf{n}$ is the normal component of $\mathbf{r}$. Using Eqs. (29), (31) and (33), the explicitly dependent terms of $H(\mathbf{x}_s, \mathbf{x})$ can be obtained as

$$\dot{H}(\mathbf{x}_s, \mathbf{x}) = -H(\mathbf{x}_s, \mathbf{x})\left[\frac{3 + \mathrm{j}kr}{r}\,\dot{r}\right] - \frac{\mathrm{e}^{-\mathrm{j}kr}}{4\pi r^3}[\mathrm{j}kr_n\dot{r} + (1 + \mathrm{j}kr)\dot{r}_n]. \tag{36}$$

Using Eqs. (33) and (36), the explicitly dependent terms of the second integrand of Eq. (2) can be obtained as

$$\{\dot{\mathbf{e}}\}^{\mathrm{T}}\{\mathbf{p}_s\} = \sum_{e=1}^{\mathrm{NE}} \iint_{\Omega_e^s} [\dot{H}(\mathbf{x}_s, \mathbf{x})p_s + \dot{J}H(\mathbf{x}_s, \mathbf{x})p_s]\,\mathrm{d}\Omega_e^s. \tag{37}$$

Again, it is unnecessary to construct the global vector $\{\dot{\mathbf{e}}\}$, as the integral can be evaluated in the element level. As with Eq. (34), the expression in Eq. (37) can be used in the calculation of $[\dot{\mathbf{A}}]\{\mathbf{p}_s\}$ in Eq. (16) by setting the observation point to the location of all boundary nodes.

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